# Diffraction and the Microscope Image

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DEUTSCHE DEMOKRATISCHE REPUBLIK
ERNST ABBE
1846 1956
VEB Carl Zeiss JENA
CARL ZEISS
JENA
10
BENGS-SKRIBELKA


CARL ZEISS
DEUTSCHE DEMOKRATISCHE REPUBLIK
1846 1956
VEB Carl Zeiss JENA
CARL ZEISS
JENA
25
BENGS-SKRIBELKA

# The Carl Zeiss Workshop 1864

Abb. 3 Ernst Abbe als Privatdozent in Jena 1863

Carl Zeiss
1863

# Some properties of wave radiation

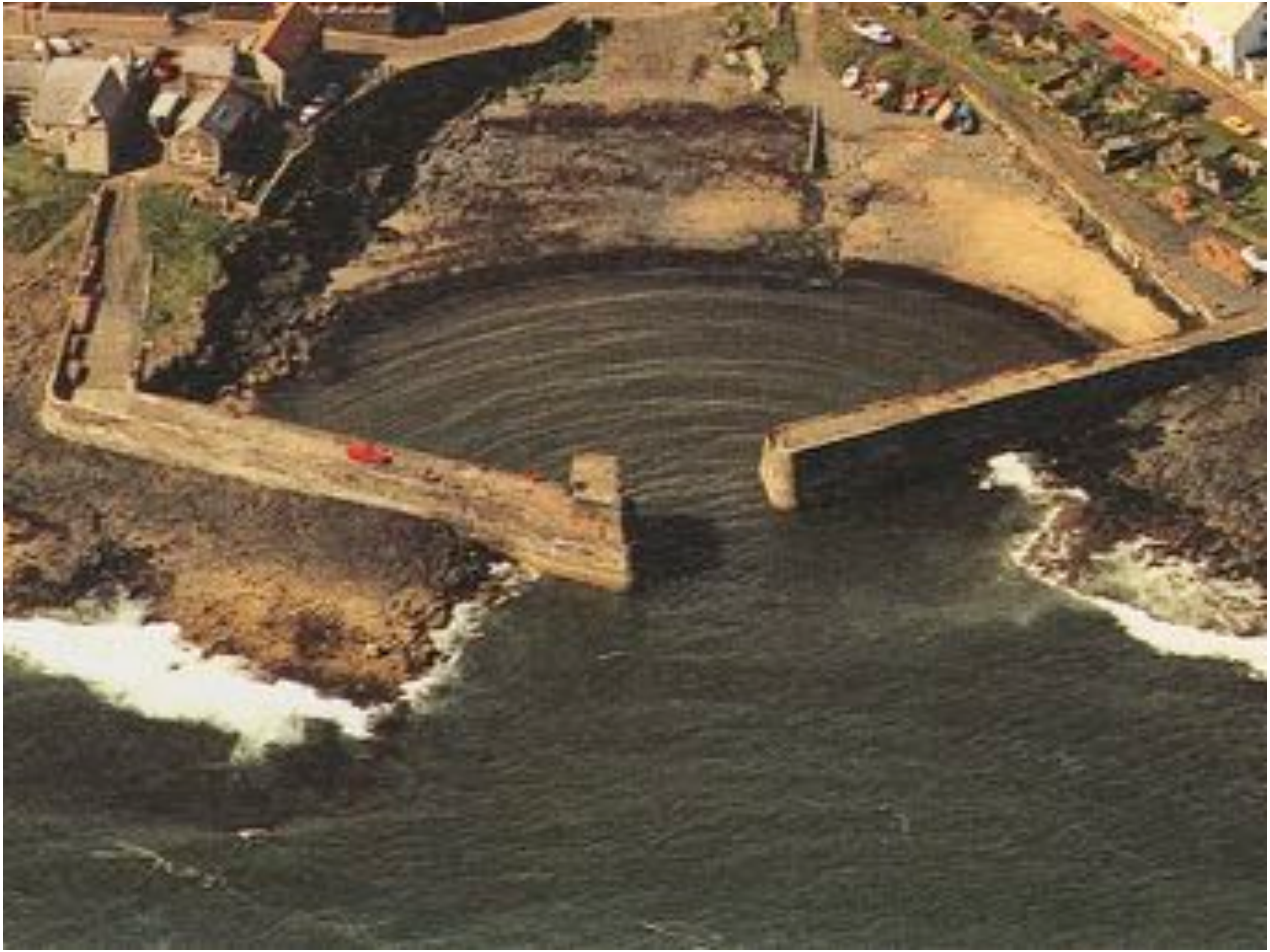
- Beams of light or electrons may be regarded as electromagnetic waves

- Waves can interfere: adding together (in certain special circumstances):

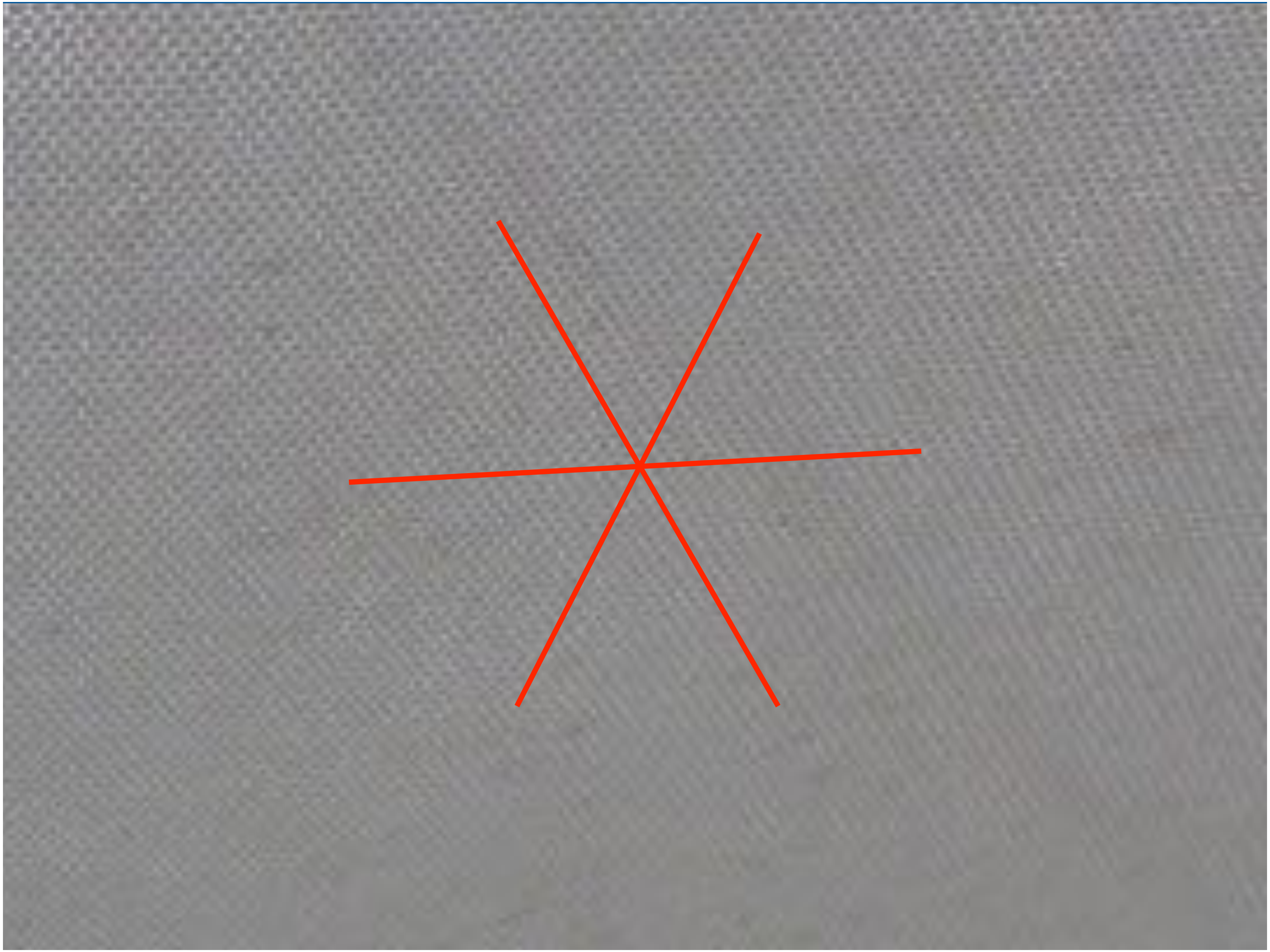
  Constructive interference – peaks correspond

  Destructive interference – peaks and troughs

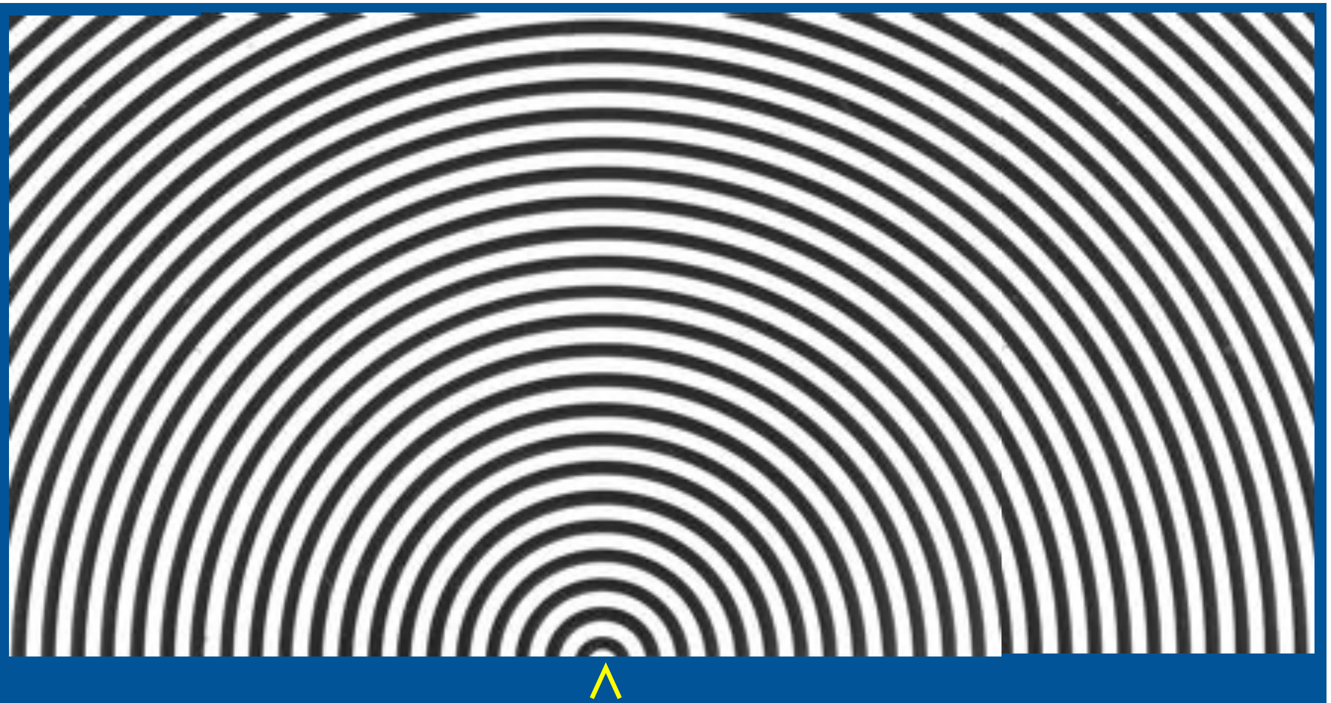
- Waves can be diffracted



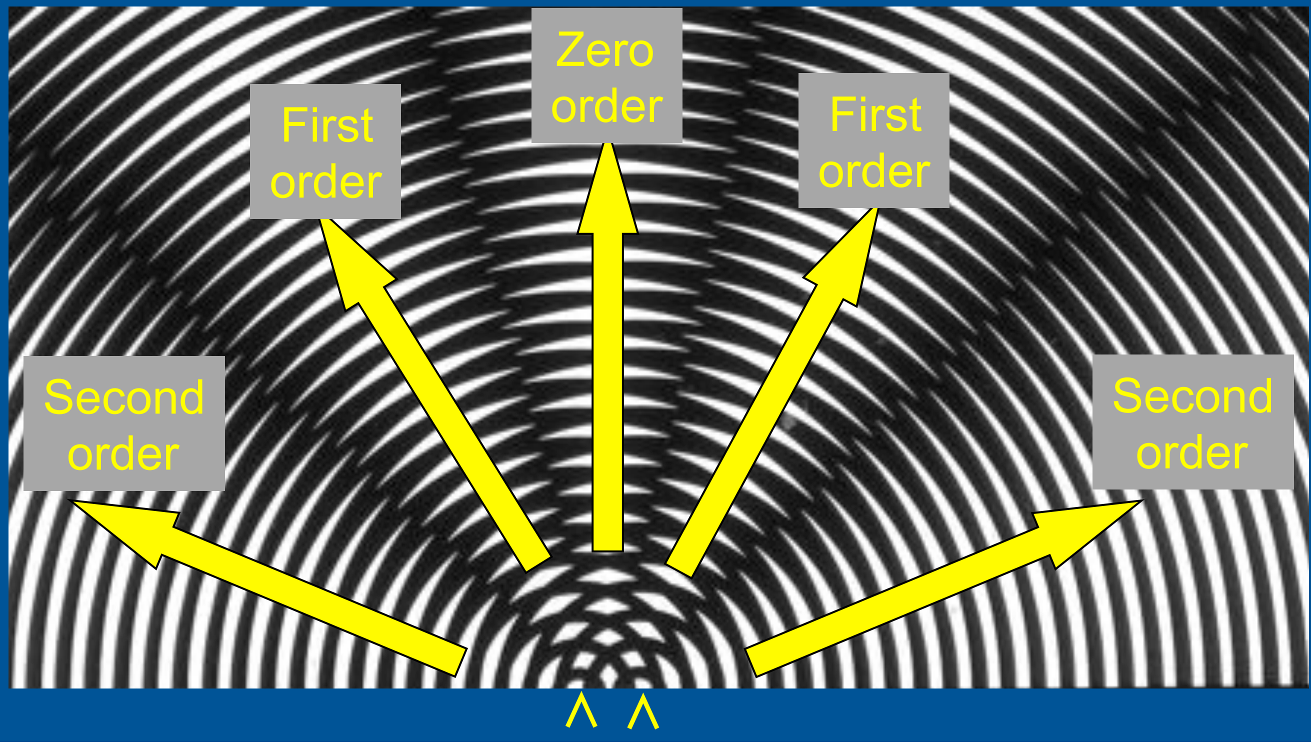
# Diffraction demonstration



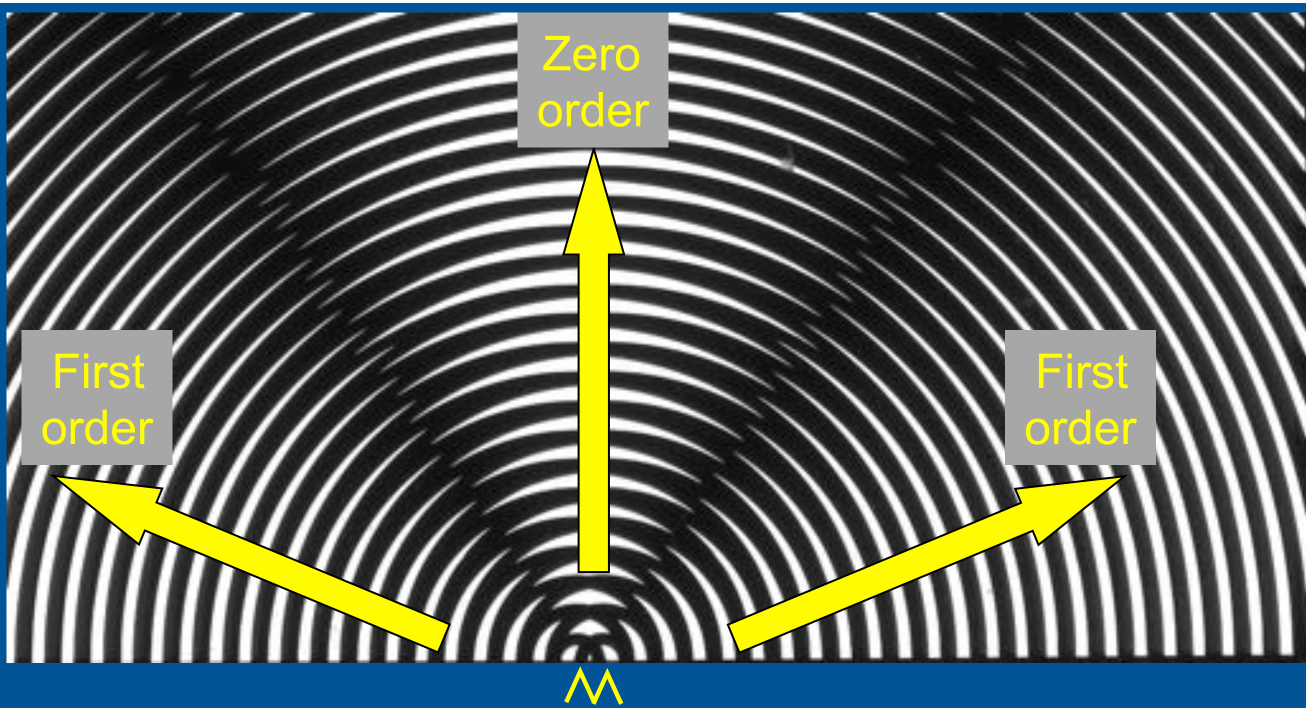
Waves radiating from a single point ∧



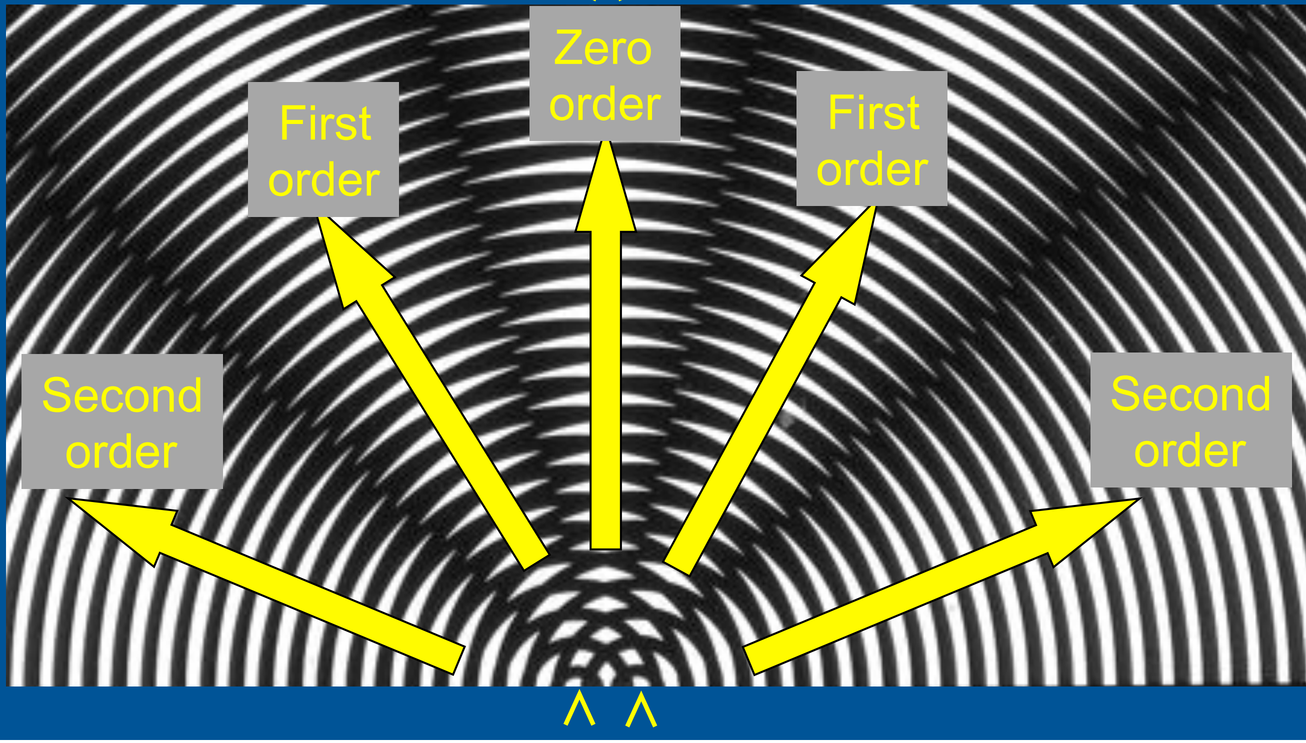
Interference between waves radiating from two points ∧ and ∧



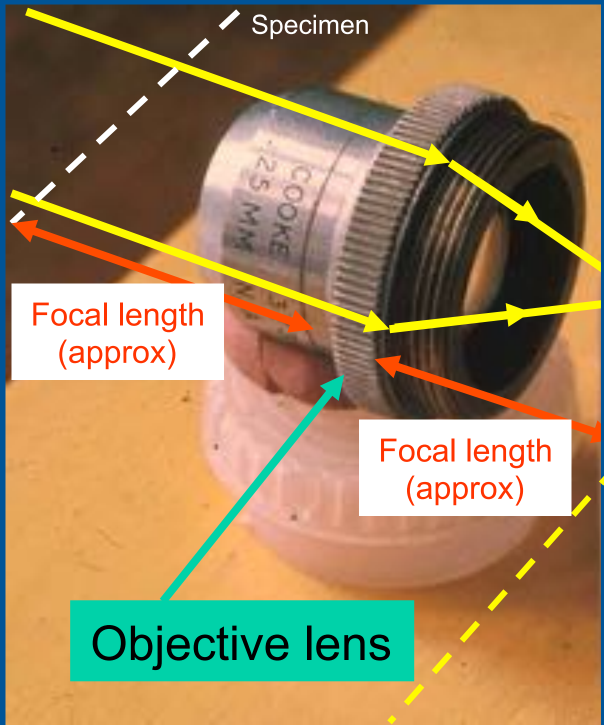
Interference between waves radiating from two more-closely-spaced points ∧ and ∧




Interference between waves radiating from two points ∧ and ∧



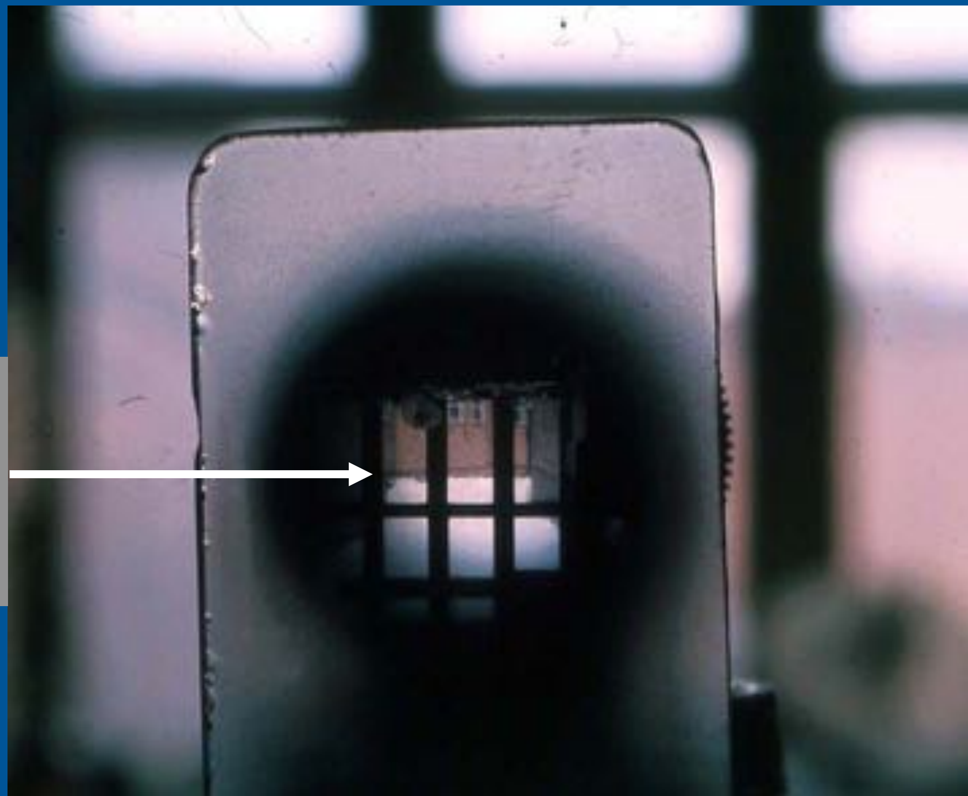
# The back focal plane of the objective

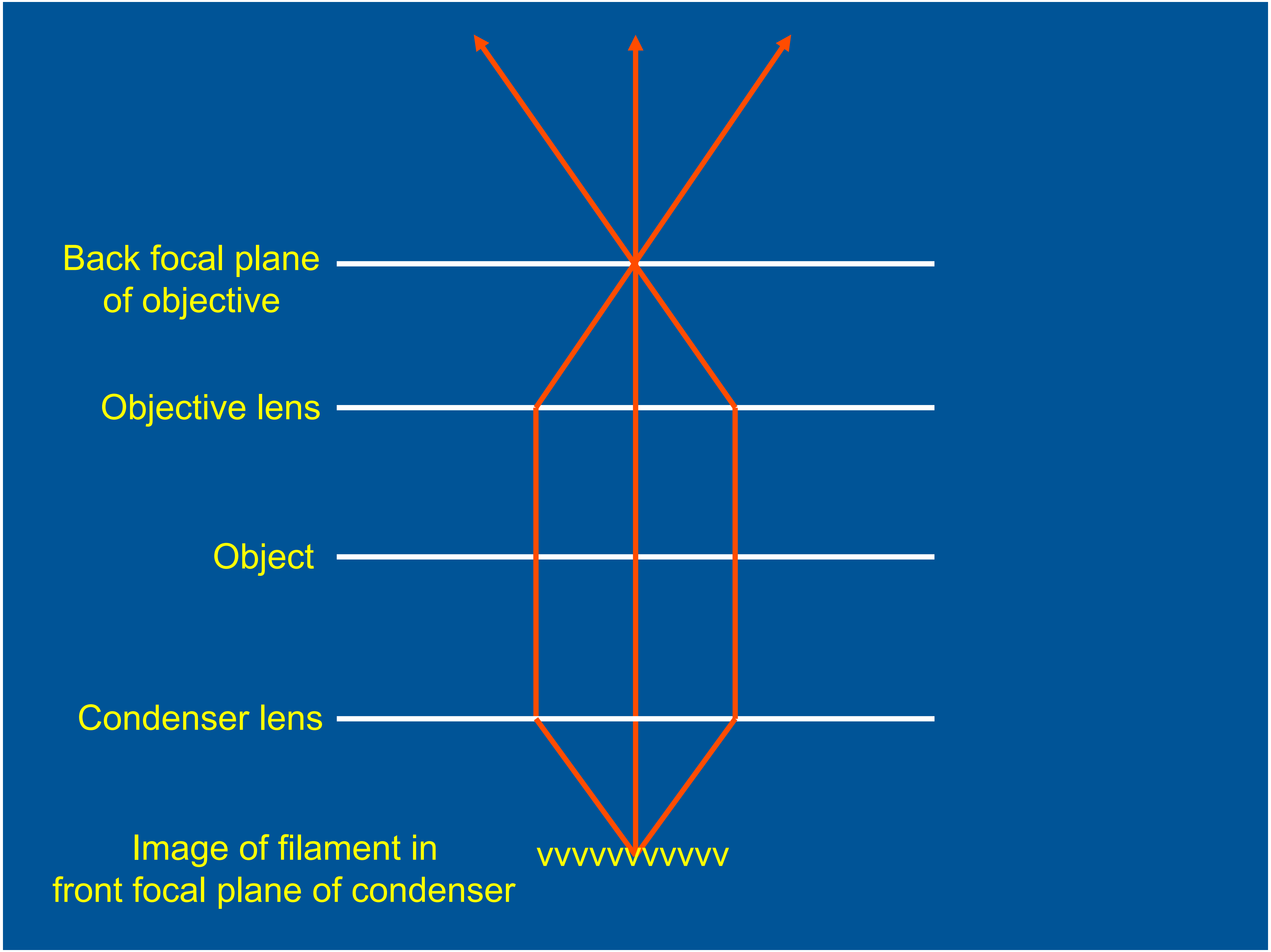


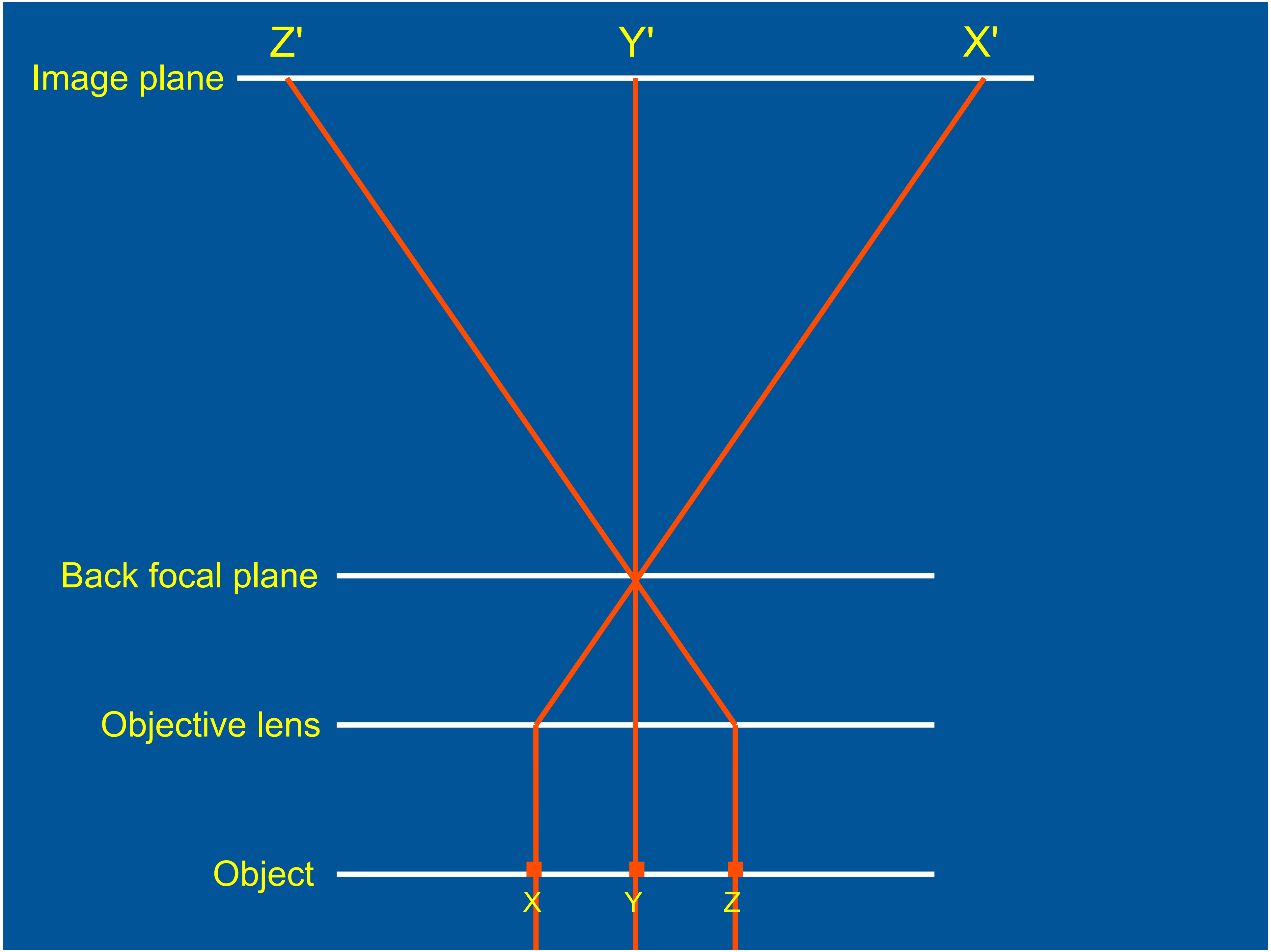

Back focal plane

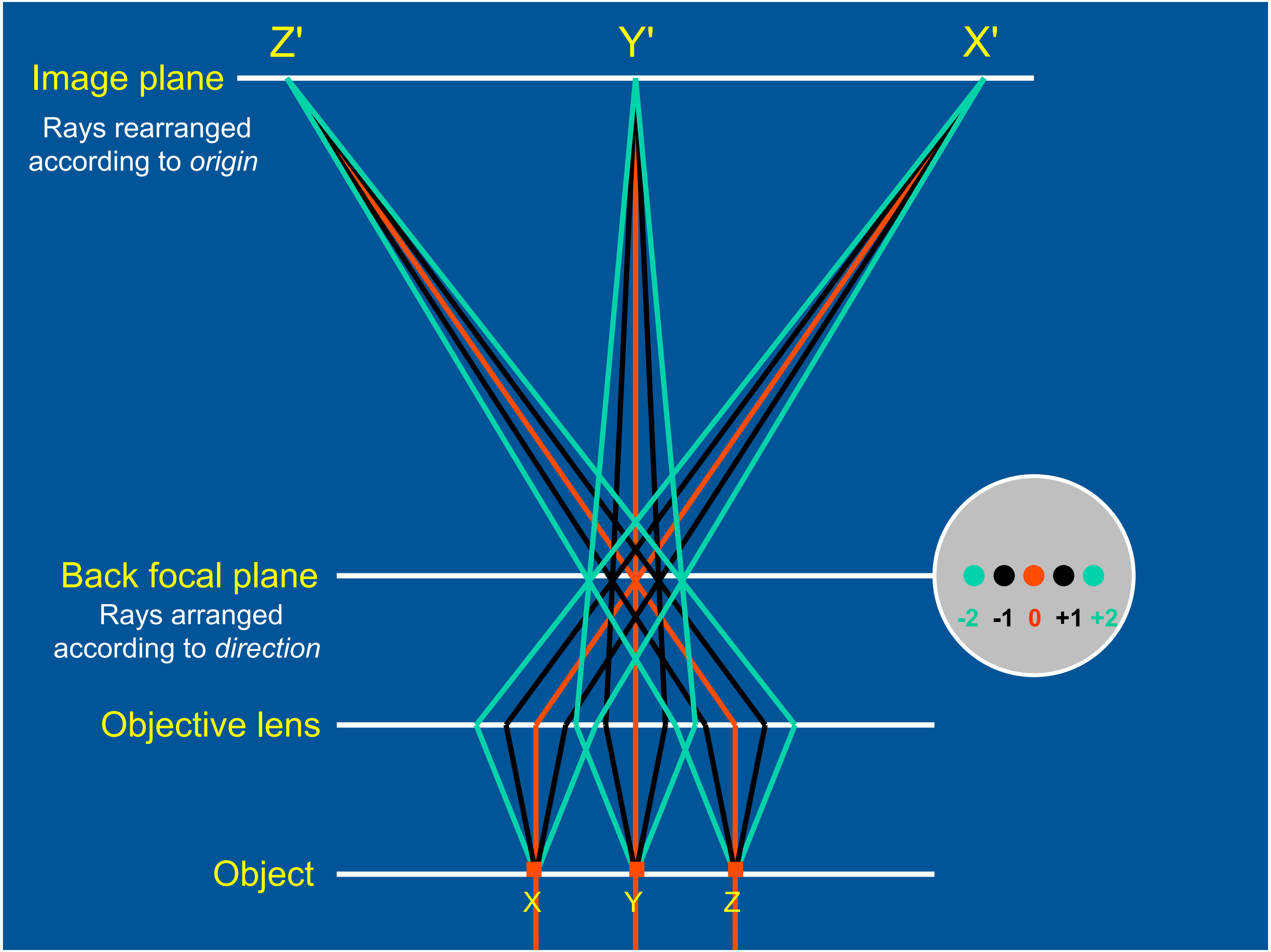
Ground glass

Image of objects at ‘infinity’
in back focal plane
of objective



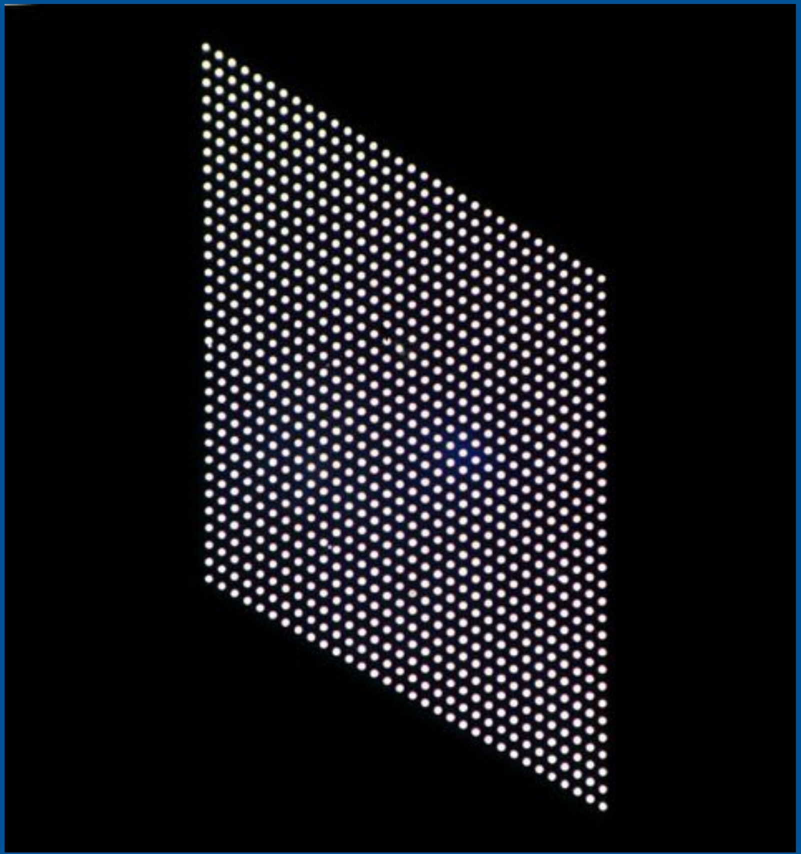

Back focal plane
of objective
Objective lens
Object
Condenser lens
Image of filament in
front focal plane of condenser
vvvvvvvvvvv


Z'
Y'
X'
Image plane
Back focal plane
Objective lens
Object
X
Y
Z

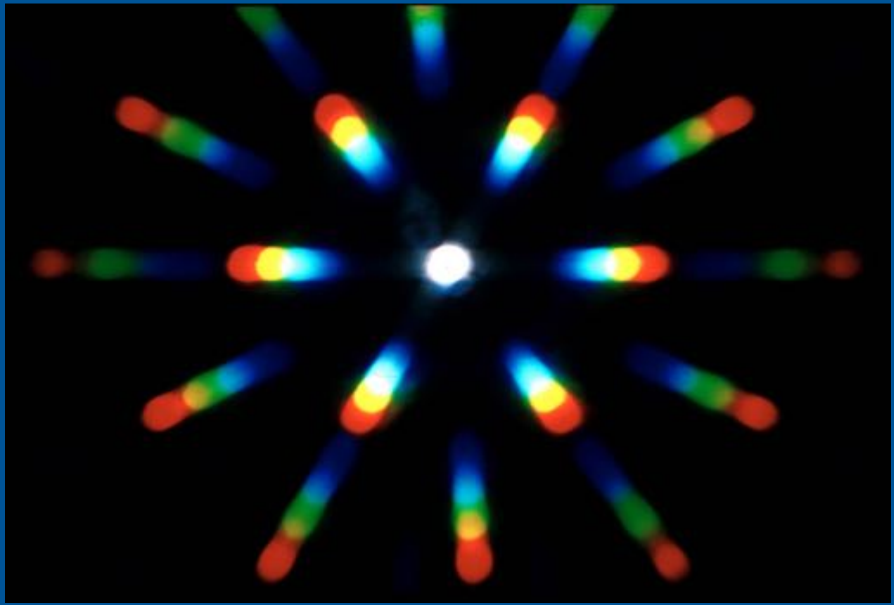

Z'
Y'
X'
Image plane
Rays rearranged according to *origin*
Back focal plane
Rays arranged according to *direction*
-2 -1 0 +1 +2
Objective lens
Object
X
Y
Z

# Diffraction in the microscope

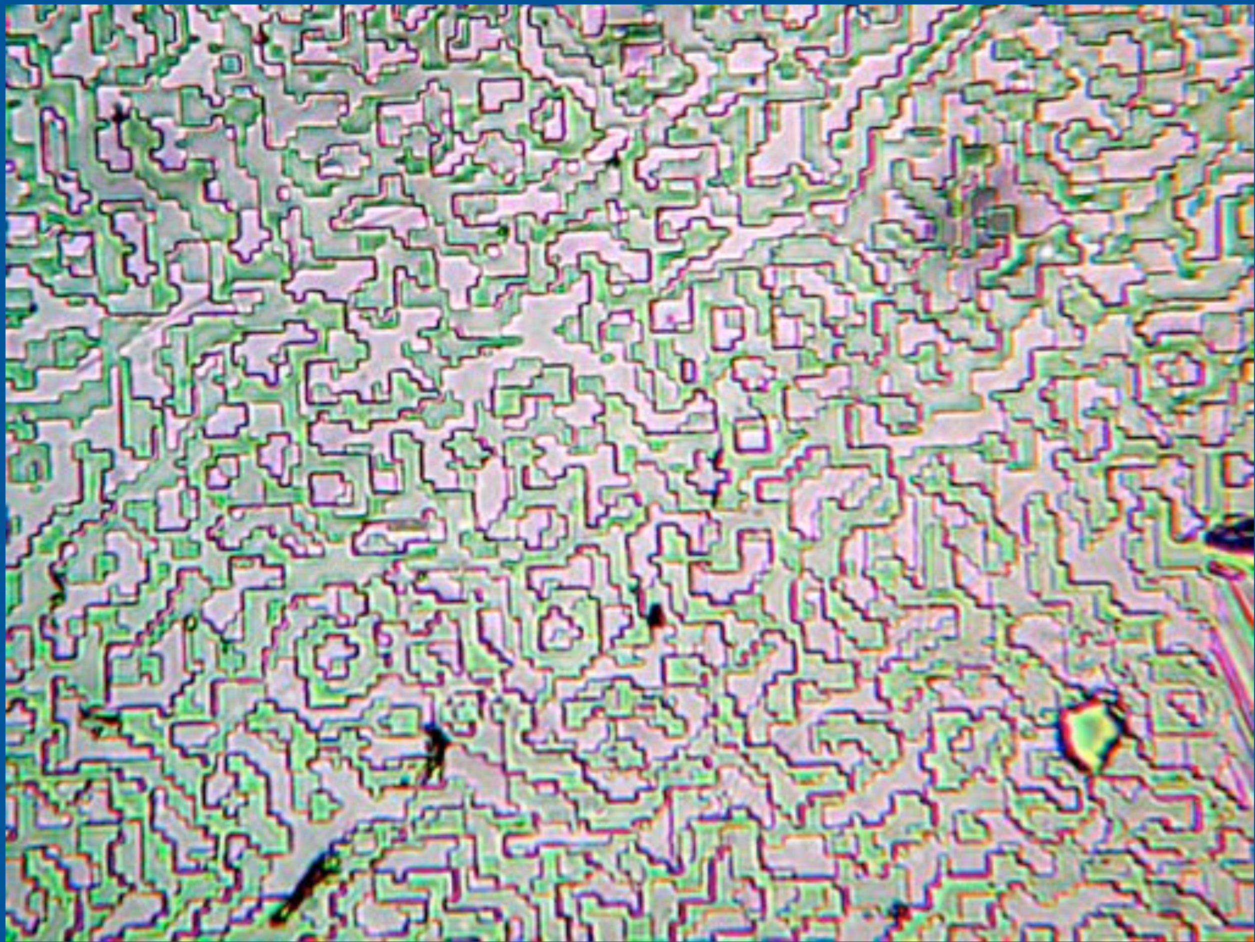
Diffraction grating



Diffraction pattern in back focal plane of objective



What will be the diffraction pattern of this grating?

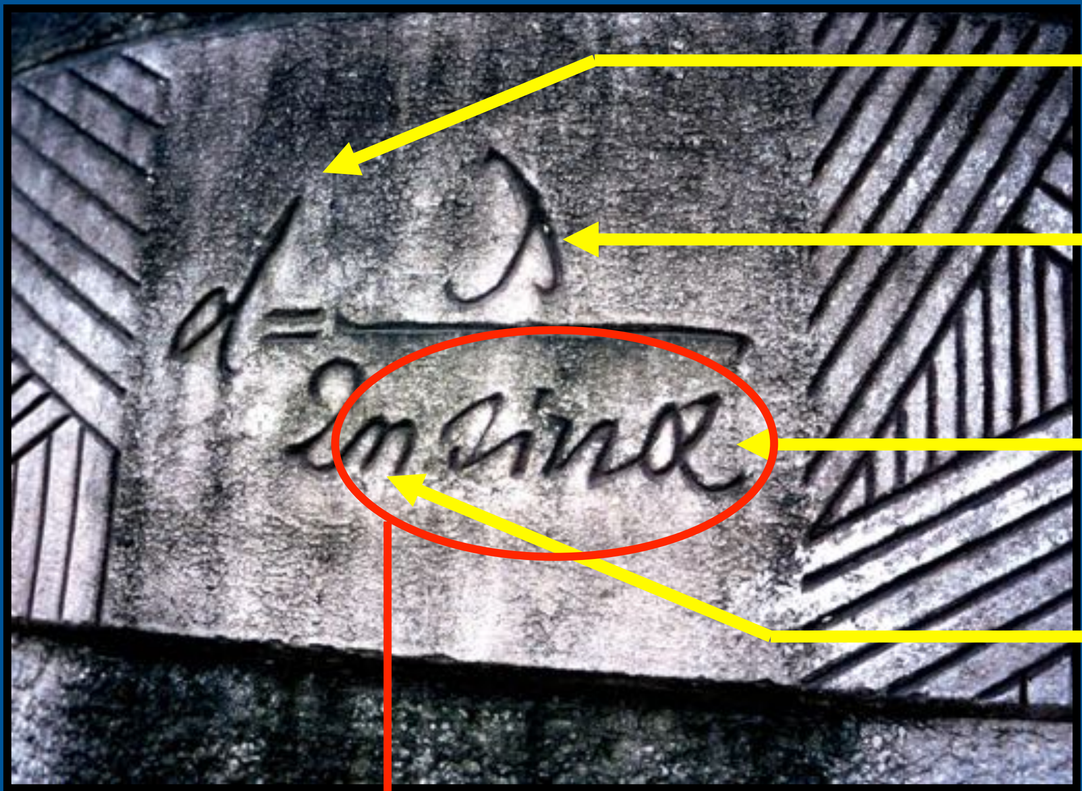


As seen in the back focal plane of the microscope in white light

# Ernst Abbe's Memorial, Jena


February1994

# Ernst Abbe’s Memorial, Jena



d — Minimum resolved distance

λ — Wavelength of imaging radiation

α — Half-aperture angle

n — Refractive index of medium

Numerical Aperture

Minimum resolved distance is now commonly expressed as

$$d = 0.61\ \lambda / NA$$

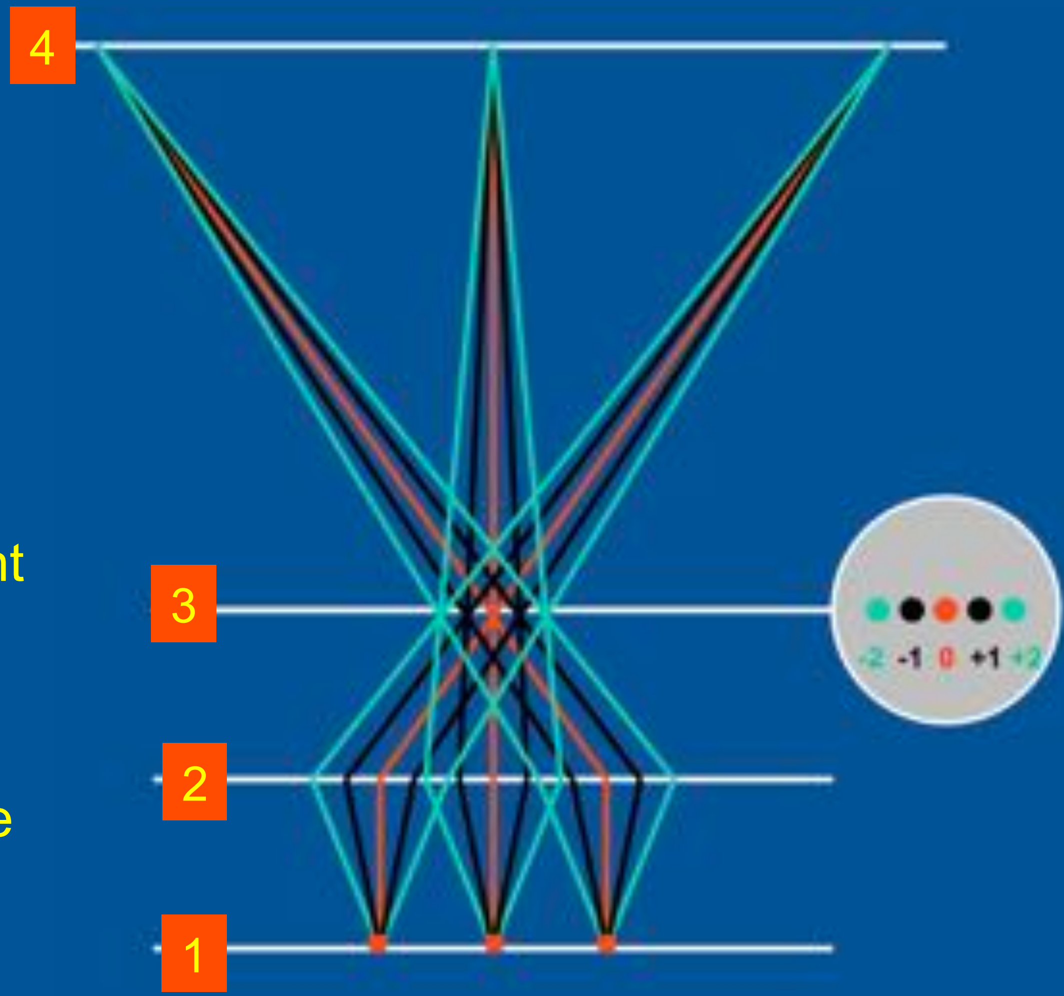
# Abbe's theory of microscopical imaging

4. Beams proceed up the microscope to the primary image plane, where they interfere to form the image.

3. Diffracted beams are brought separately to focus in the back focal plane of the objective

2. Some – but not all – of these diffracted beams enter the objective

1. The object diffracts light – finer detail more obliquely than coarser



From this theorem 2), in connection with 1) is to be inferred: the linear distance of the diffraction-spectra, which appear in the back-focal-plane of the objective is always $= \frac{\lambda}{\delta} \cdot f$, if corresponding points in every two consecutive spectra are considered — independent of the inclination of the incident rays to the grating. — If you go from central light to oblique light, all the spectra move within the back-plane of the system, without changing their relative position.

3). All the rays, which result by diffraction, from one incident ray have their oscillations in equal phase, if points are compared on these rays which are situated in the back-focal-plane, where the spectra are formed as images of the illuminating object; all those rays therefore must interfere ~~in~~ within the planes where they ~~are~~ meet — that is the plane, ~~of the images~~ where an image of the grating is formed by the objectif (the conjugate focus to the microscopic object)


4) If Δ be the linear distance of the 2 interfering rays in the back-focal-plane, l the distance of the conjugate focus to the object (= length of tube of the microscope) the maxima and minima of light, resulting by interference in the plane C, have a distance

$$\delta' = l \cdot \frac{\lambda}{\Delta}$$

Now if the two rays considered are consecutive rays from a grating with the distance δ, Δ is $= \frac{\lambda}{\delta} \cdot f$; therefore

$$\delta' = \delta \cdot \frac{l}{f}$$

— that is the same distance, in which the lines of the grating would appear in a purely dioptrical image, under the same circumstances.

Ernst Abbe to J. W. Stephenson
15 December 1876

I shall be very glad, if you should like to show the experiments to the Microscopical Society – especially if you should think it convenient to produce them not as paradox phenomena, but rather as phenomena illustrating a distinct idea of the functions of the microscope. For there is no want of optical curiosities among microscopists; and I take no interest in bringing forth more of them. – Please, make any use of my explanations, you like.

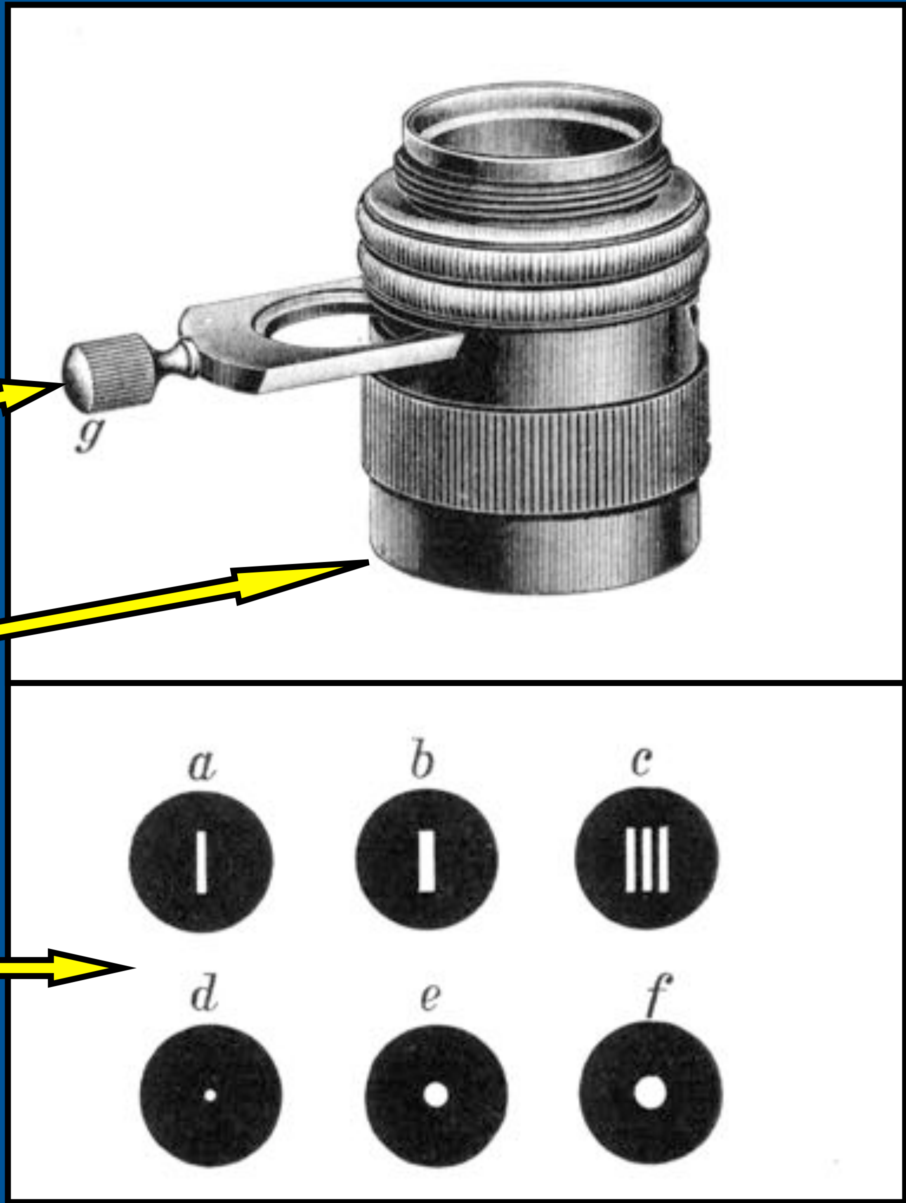
Ernst Abbe to J. W. Stephenson 15 December 1876

# Abbe's Diffraction Apparatus

'Drawer' at level of back focal plane

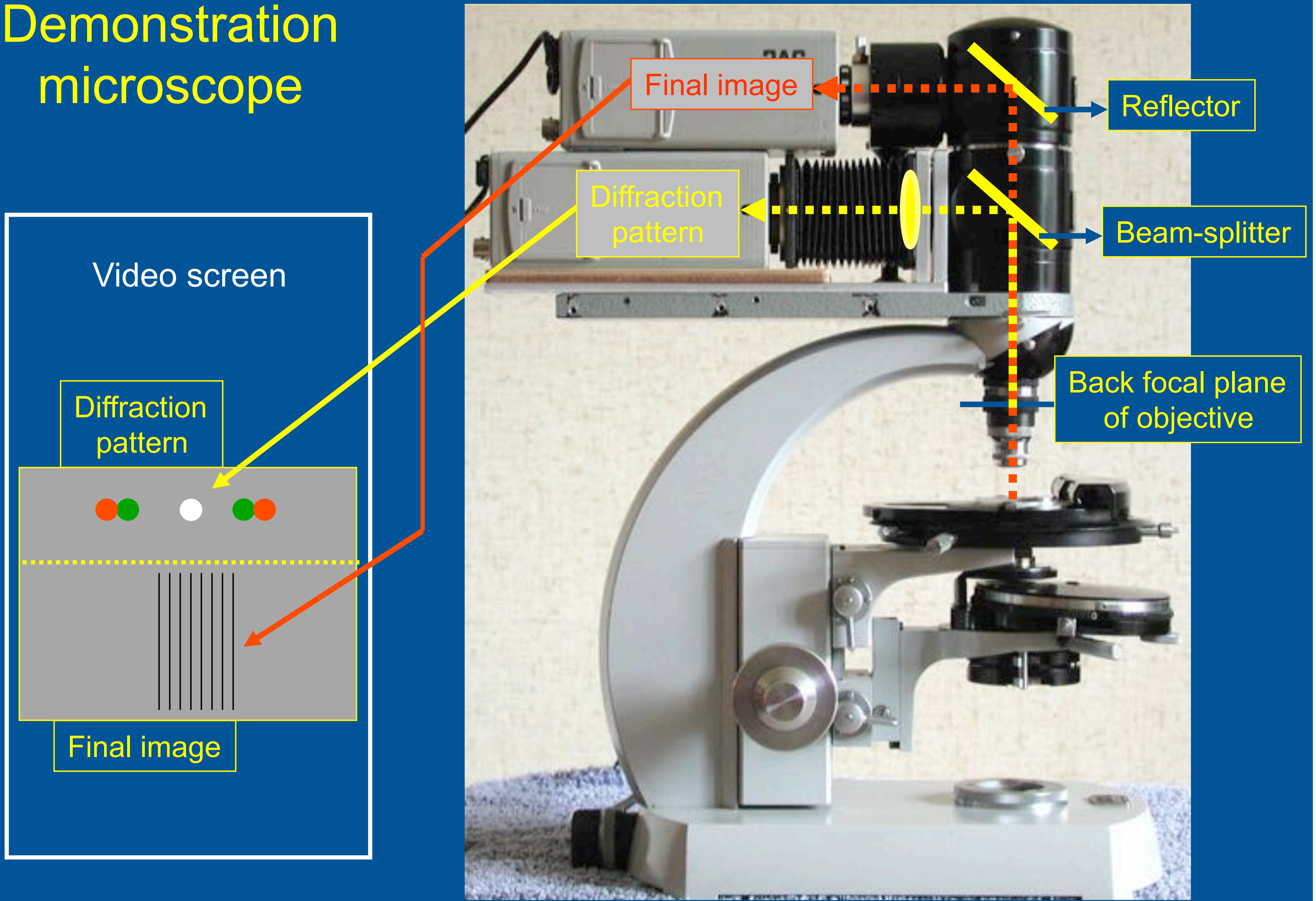
Screw thread for objective lens

Masks for insertion into back focal plane



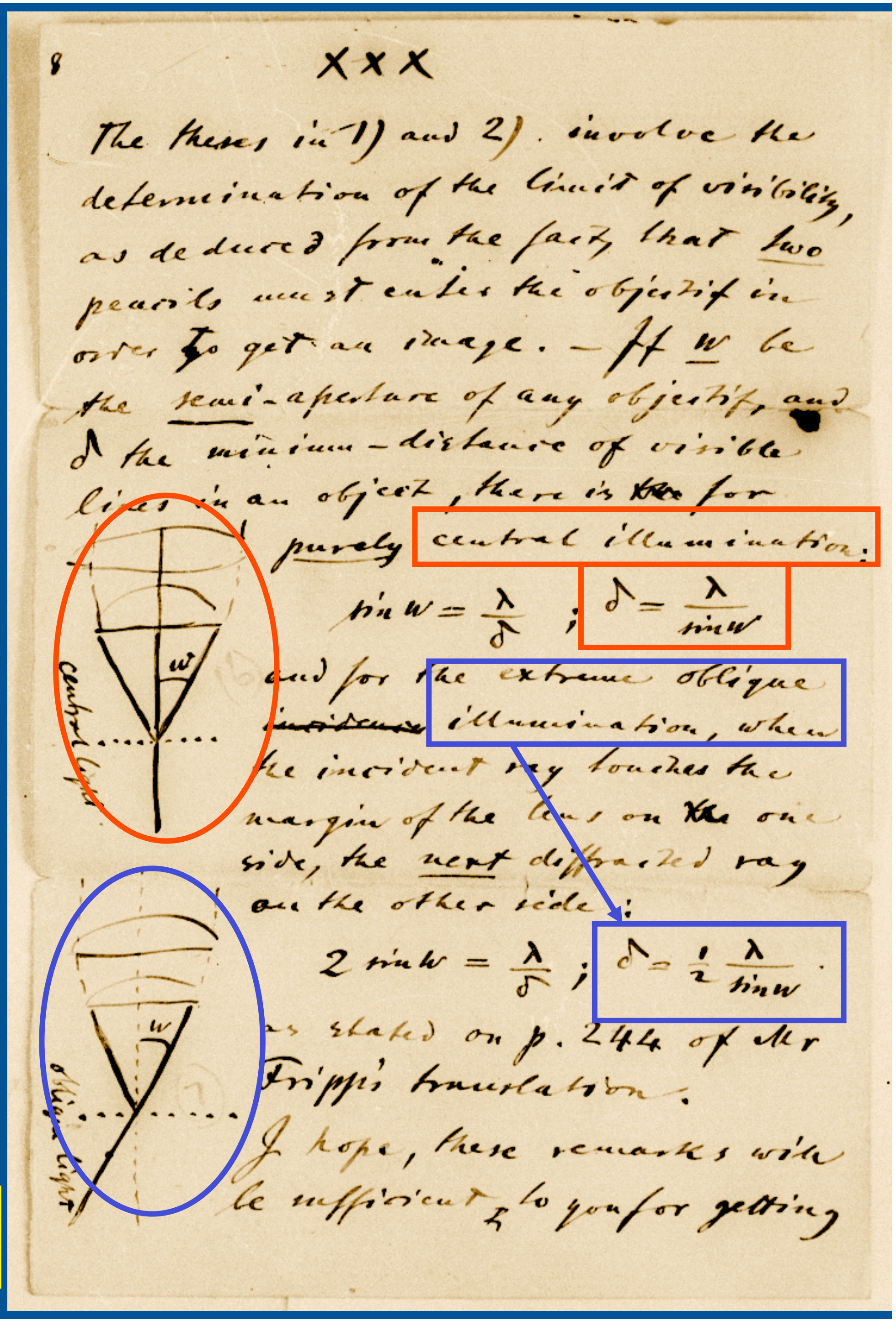

DIFFRACTIONSPLATTE
NACH ABBE
C. ZEISS JENA

# Demonstration microscope



# Demonstration of Abbe’s Diffraction Experiments


# Abbe's explanation of the advantage of a full illuminating aperture



8 XXX

The theses in 1) and 2). involve the determination of the limit of visibility, as deduced from the fact that two pencils must enter the objectif in order to get an image. — If w be the semi-aperture of any objectif, and δ the minimum-distance of visible lines in an object, there is for purely central illumination:

$$\sin w = \frac{\lambda}{\delta} ; \quad \delta = \frac{\lambda}{\sin w}$$

and for the extreme oblique ~~incidence~~ illumination, when the incident ray touches the margin of the lens on ~~the~~ one side, the next diffracted ray on the other side:

$$2 \sin w = \frac{\lambda}{\delta} ; \quad \delta = \frac{1}{2} \frac{\lambda}{\sin w}.$$

as stated on p. 244 of Mr Fripp's translation.

I hope, these remarks will be sufficient to you for getting



Ernst Abbe to J. W. Stephenson
15 December 1876

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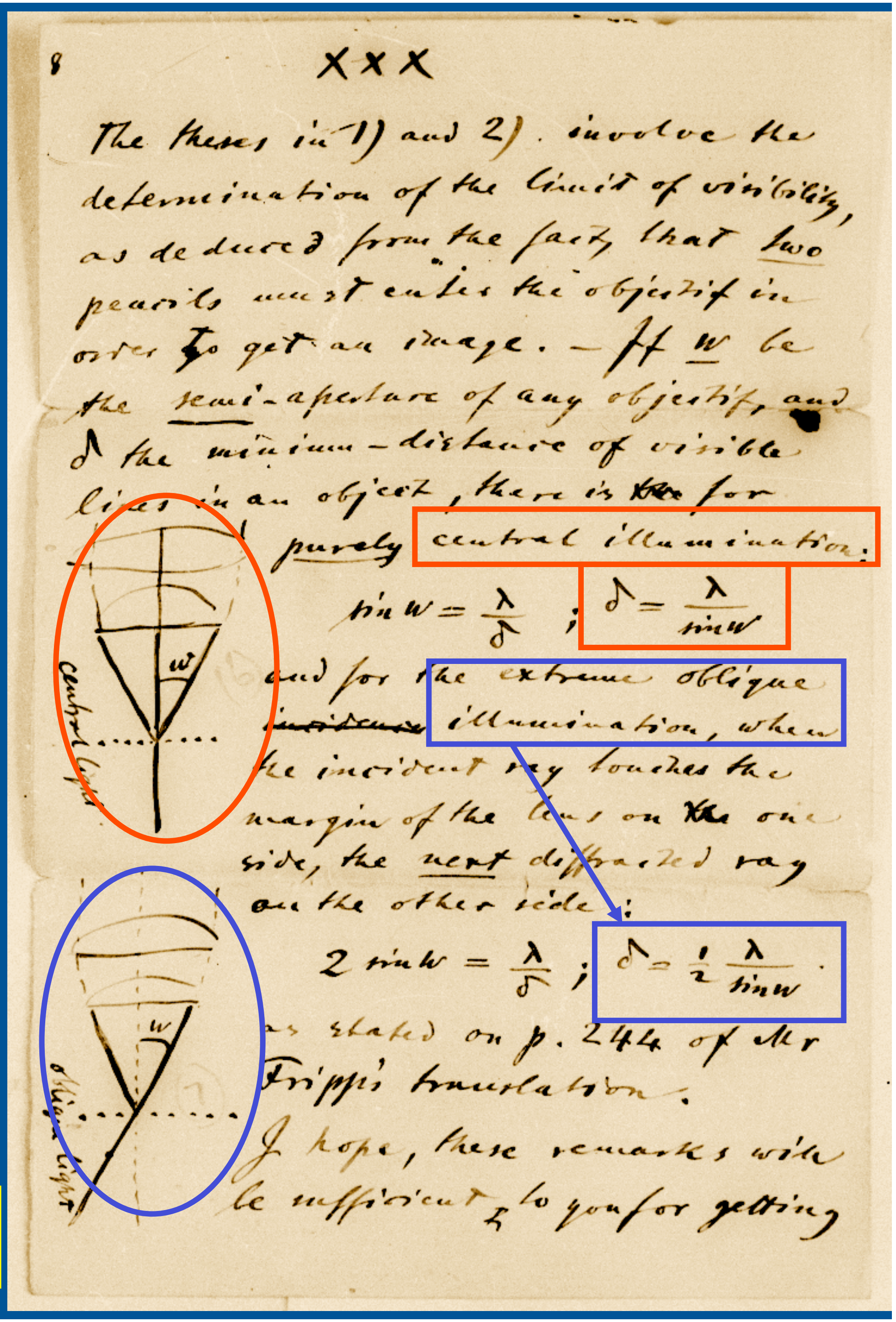

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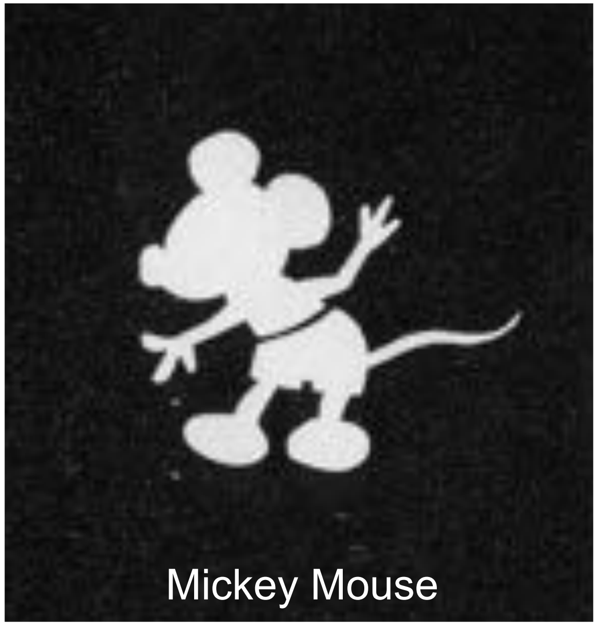
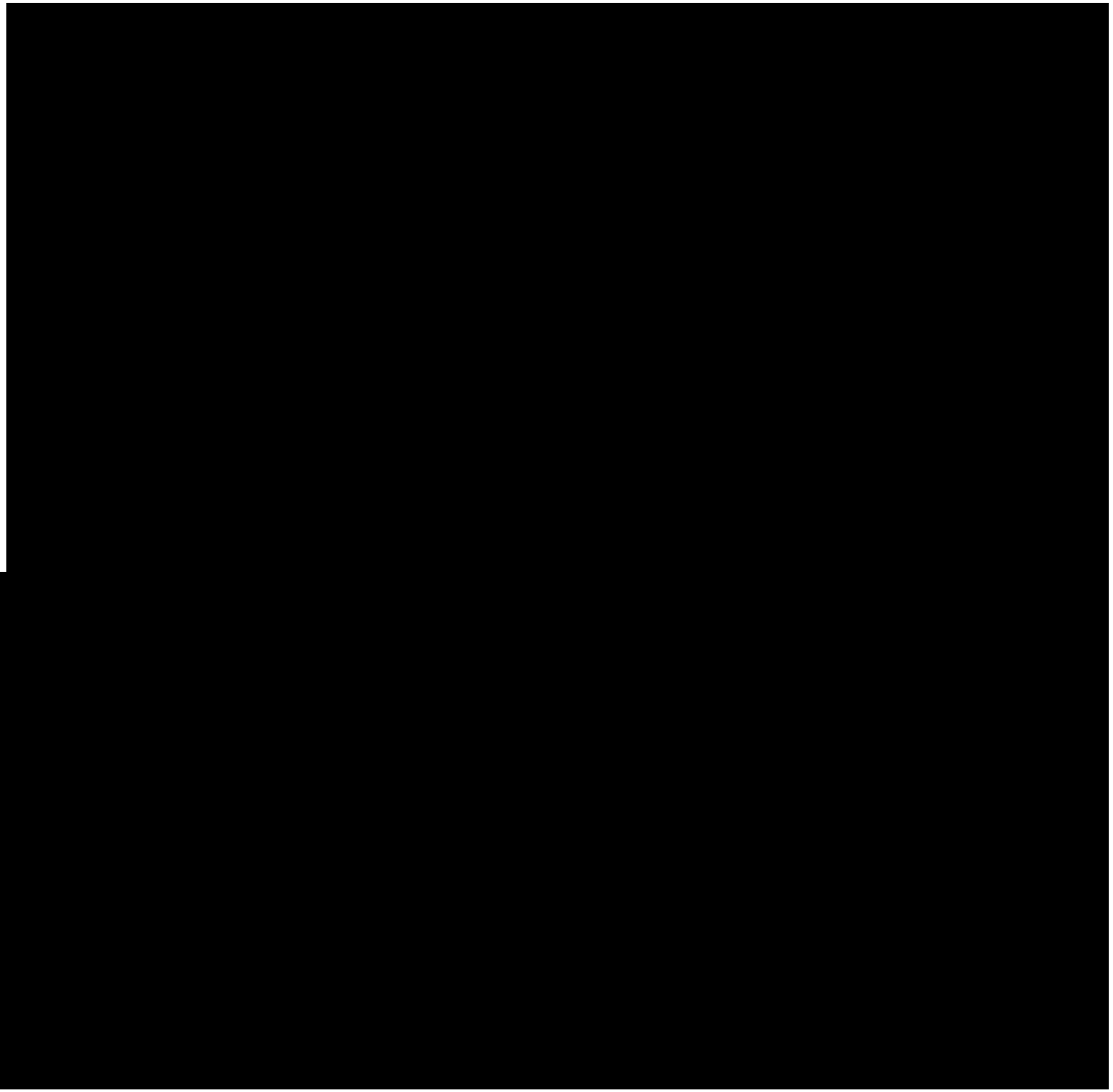
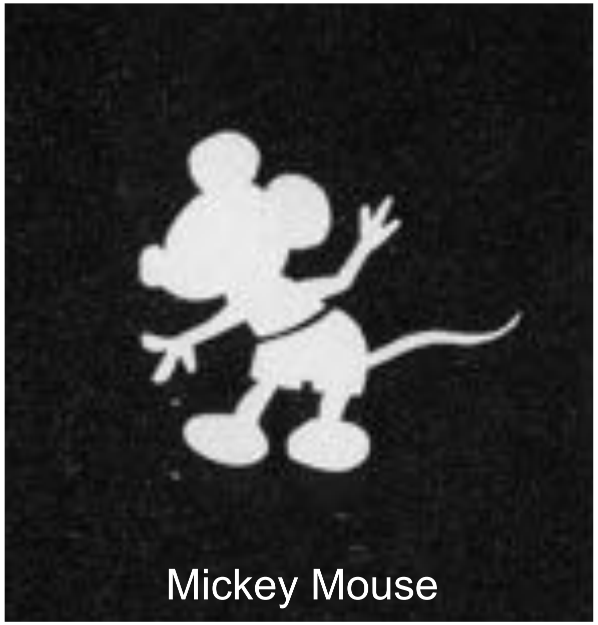


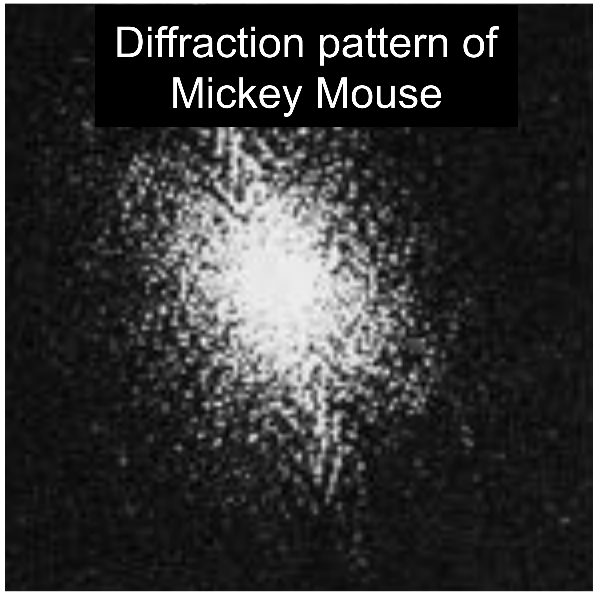

Diffraction pattern of
Mickey Mouse

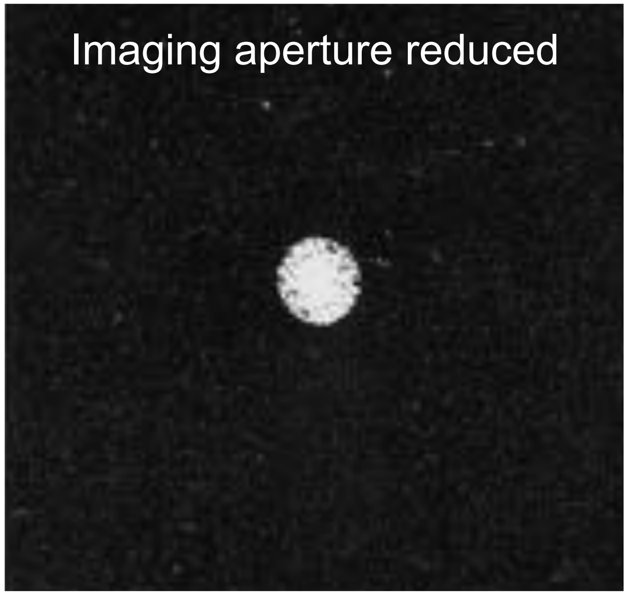


From Harburn, Taylor & Welberry: *Atlas of Optical Transforms*





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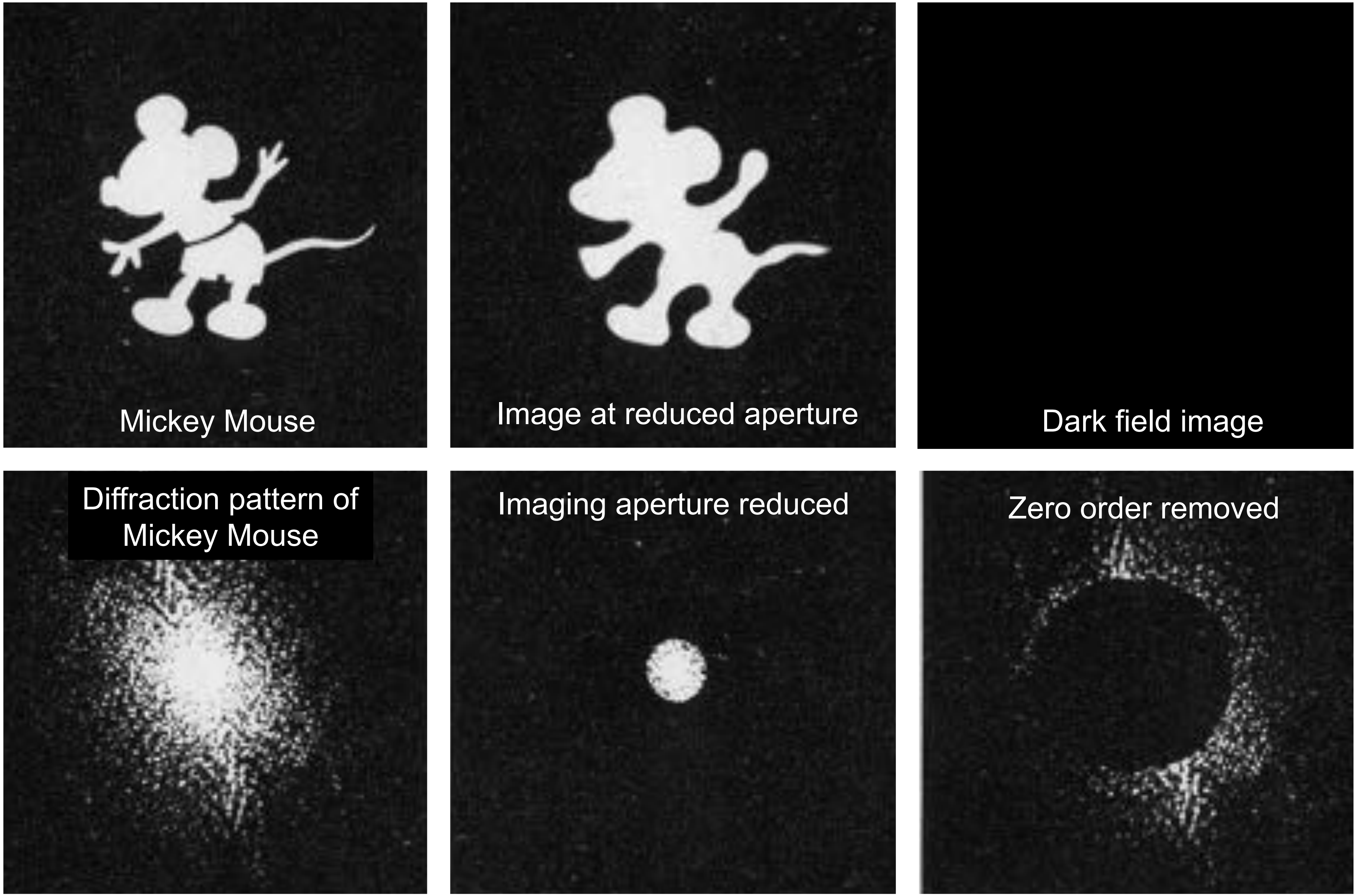



From Harburn, Taylor & Welberry: *Atlas of Optical Transforms*

# Do it yourself?

- Light source:
  - Remove condenser
  - Close illuminated field diaphragm

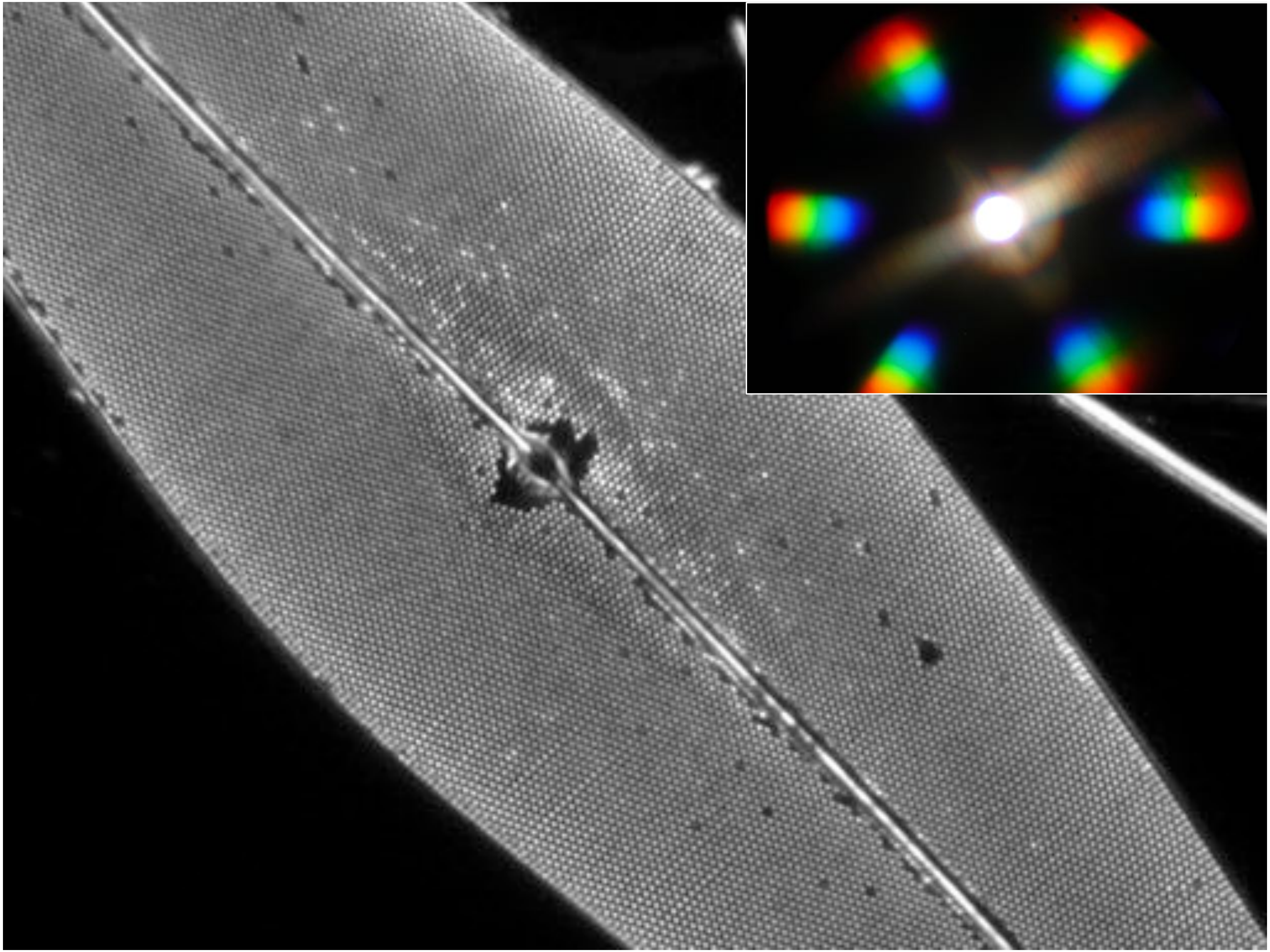
    Provides almost a point source, almost at infinity

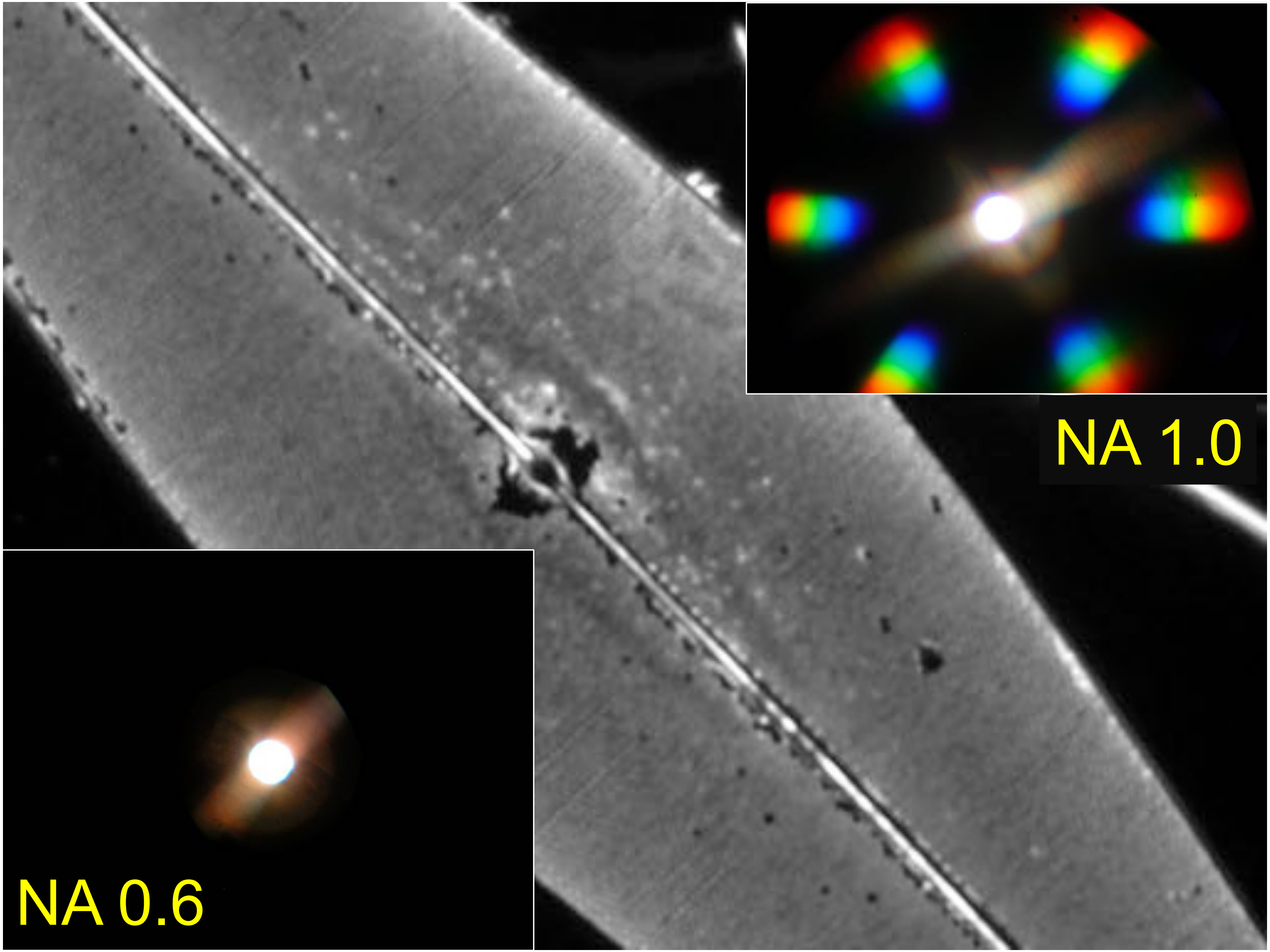
- To see diffraction pattern in back focal plane:
  - Pinhole eyepiece, or
  - Telescope, or
  - Bertrand lens

- Objective:
  - Several of different numerical apertures to suit specimen fine detail
  - With iris diaphragm

- Specimen:
  - Diatom
  - Stage micrometer
  - CD (commercial, not writable, viewed from unprinted top side with 40/0.65)

0.6
1.0
Planapo 40/1.0
160/-
Planapo 63/
160/-




NA 1.0
NA 0.6